

## Constraints on unparticle interactions from invisible decays of $Z$ , quarkonia and neutrinos

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**ABSTRACT:** Unparticles ( $\mathcal{U}$ ) interact weakly with particles. The direct signature of unparticles will be in the form of missing energy. We study constraints on unparticle interactions using totally invisible decay modes of  $Z$ , vector quarkonia  $V$  and neutrinos. The constraints on the unparticle interaction scale  $\Lambda_{\mathcal{U}}$  are very sensitive to the dimension  $d_{\mathcal{U}}$  of the unparticles. From invisible  $Z$  and  $V$  decays, we find that with  $d_{\mathcal{U}}$  close to 1 for vector  $\mathcal{U}$ , the unparticle scale  $\Lambda_{\mathcal{U}}$  can be more than  $10^4$  TeV, and for  $d_{\mathcal{U}}$  around 2, the scale can be lower than one TeV. From invisible neutrino decays, we find that if  $d_{\mathcal{U}}$  is close to  $3/2$ , the scale can be more than the Planck mass, but with  $d_{\mathcal{U}}$  around 2 the scale can be as low as a few hundred GeV. We also study the possibility of using  $V(Z) \rightarrow \gamma + \mathcal{U}$  to constrain unparticle interactions, and find that present data give weak constraints.

**KEYWORDS:** Neutrino Physics, Beyond Standard Model, Heavy Quark Physics.

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**1. Introduction**

Recently Georgi proposed an interesting idea to describe possible scale invariant effects at low energies by an operator  $O_{\mathcal{U}}$ , termed unparticle [1]. Based on a specific scale invariant theory with a non-trivial infrared fixed point by Banks and Zaks [2], it was argued that operators  $O_{\text{BZ}}$  made of BZ fields may interact with operators  $O_{\text{SM}}$  made of Standard Model (SM) fields at some high energy scale by exchanging particles with large masses,  $M_{\mathcal{U}}$ , and induce interactions of the form

$$\frac{\tilde{C}_{\mathcal{U}}}{M_{\mathcal{U}}^{d_{\text{SM}}+d_{\text{BZ}}-4}} O_{\text{SM}} O_{\text{BZ}}, \tag{1.1}$$

where  $d_{\text{BZ}}$  and  $d_{\text{SM}}$  are the dimensions of the operators  $O_{\text{BZ}}$  and  $O_{\text{SM}}$ .

At another scale  $\Lambda_{\mathcal{U}}$  the BZ sector induces dimensional transmutation, and below that scale the BZ operator  $O_{\text{BZ}}$  matches on to unparticle operator  $O_{\mathcal{U}}$  with dimension  $d_{\mathcal{U}}$ . The unparticle interaction with SM particles at low energy then has the form

$$C_{\mathcal{U}} \lambda \Lambda_{\mathcal{U}}^{4-d_{\text{SM}}-d_{\mathcal{U}}} O_{\text{SM}} O_{\mathcal{U}}, \quad \lambda = \left( \frac{\Lambda_{\mathcal{U}}}{M_{\mathcal{U}}} \right)^{d_{\text{SM}}+d_{\text{BZ}}-4}. \tag{1.2}$$

The unparticle may have different Lorentz structures such as a scalar  $O_{\mathcal{U}}$ , a vector  $O_{\mathcal{U}}^{\mu}$ , a spinor  $O_{\mathcal{U}}^s$ , and etc. . . The specific form of SM particle and unparticle interactions are not known and are usually parameterized in terms of operators. In ref. [17] a class of operators involving SM particles and unparticles are listed. Using these operators one can study unparticle phenomenology in a systematic way.

One of the main phenomenological goals of unparticle physics study is to find out at what energy scale unparticle effects may show up [1, 3–27]. The most direct signatures of unparticles will be in the form of missing energy in invisible decays of particles with an unparticle  $\mathcal{U}$  in the final state. In this paper we study constraints on the unparticle interactions using  $Z \rightarrow \text{invisible}$ ,  $V \rightarrow \text{invisible}$ , and  $\nu \rightarrow \text{invisible}$  decays. We will also study the possibility of using  $V(Z) \rightarrow \gamma + \mathcal{U}$  to constrain unparticle interactions.

## 2. Constraints from invisible decay of $Z$ boson

The process  $Z \rightarrow \mathcal{U}$  contributes to invisible decay of  $Z$ . For this process the following operators, with SM fields and derivatives contribute less than or equal to 4 dimensions, will contribute [17]

$$\lambda'_{\text{bO}} \Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}} B_{\mu\nu} \partial^\mu O_{\mathcal{U}}^\nu, \quad \tilde{\lambda}'_{\text{bO}} \Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}} \tilde{B}_{\mu\nu} \partial^\mu O_{\mathcal{U}}^\nu, \quad \lambda'_{\text{hh}} \Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}} (H^\dagger D_\mu H) O_{\mathcal{U}}^\mu. \quad (2.1)$$

Here the vector unparticle operator  $O_{\mathcal{U}}^\mu$  is hermitian and transverse with,  $\partial_\mu O_{\mathcal{U}}^\mu = 0$ .

The matrix elements for  $Z \rightarrow \mathcal{U}^\mu$  resulting from the above operators are given by [17]

$$\begin{aligned} M(Z \rightarrow \mathcal{U}^\mu, \lambda'_{\text{bO}}) &= \lambda'_{\text{bO}} \Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}} \sin \theta_W (k_Z \cdot k_O \epsilon_Z \cdot \epsilon_O - k_Z \cdot \epsilon_O k_O \cdot \epsilon_Z), \\ M(Z \rightarrow \mathcal{U}^\mu, \tilde{\lambda}'_{\text{bO}}) &= i \tilde{\lambda}'_{\text{bO}} \Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}} \sin \theta_W \epsilon_{\mu\nu\alpha\beta} k_Z^\mu \epsilon_Z^\nu k_O^\alpha \epsilon_O^\beta, \\ M(Z \rightarrow \mathcal{U}^\mu, \lambda'_{\text{hh}}) &= -2 \text{Im}(\lambda'_{\text{hh}}) \Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}} \frac{e}{\sin(2\theta_W)} \frac{v^2}{2} \epsilon_Z \cdot \epsilon_O, \end{aligned} \quad (2.2)$$

where  $v = 246$  GeV is the vacuum expectation value of the Higgs doublet. Since  $k_Z = k_O$  for  $Z$  to  $O_{\mathcal{U}}^\mu$  transition, the second term in the above does not contribute. Here we have used the notation  $\epsilon_Z$  and  $\epsilon_O$  to describe the polarizations of  $Z$  and  $\mathcal{U}^\mu$ .

For a decay of a particle into an unparticle and other particles, the differential decay rate is given by [1]

$$d\Gamma(P \rightarrow \mathcal{U}) = \frac{|\overline{M}|^2}{2m_P} d\Phi(P), \quad (2.3)$$

where  $d\Phi(P)$  is the phase space factor for the decay. It is given by

$$d\Phi = \int (2\pi)^4 \delta^4(P - \sum_j p_j) \prod_j d\Phi(p_j) \frac{d^4 p_j}{(2\pi)^4}. \quad (2.4)$$

For a particle the phase factor  $d\Phi(p_j)$  is equal to  $2\pi \theta(p_j^0) \delta(p_j^2 - m_j^2)$ , and for an unparticle it is given by  $A_{d_{\mathcal{U}}} \theta(p^0) \theta(p^2) (p^2)^{d_{\mathcal{U}}-2}$  with  $A_{d_{\mathcal{U}}} = (16\pi^{5/2} / (2\pi)^{2d_{\mathcal{U}}}) (\Gamma(d_{\mathcal{U}} + 1/2) / \Gamma(d_{\mathcal{U}} - 1) \Gamma(2d_{\mathcal{U}}))$ .

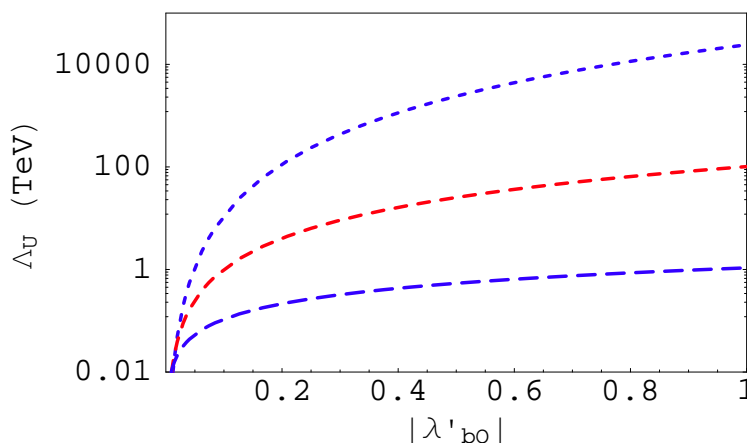
The decay width of a particle decay into an unparticle is given by

$$\Gamma(P \rightarrow \mathcal{U}) = \frac{|\overline{M}|^2}{2m_P} A_{d_{\mathcal{U}}} (m_P^2)^{d_{\mathcal{U}}-2}. \quad (2.5)$$

We note that in the limiting case of  $d_{\mathcal{U}} = 1$  the decay width becomes zero since  $A_{d_{\mathcal{U}}}$  has a factor  $1/\Gamma(d-1)$  which goes to zero when  $d_{\mathcal{U}} \rightarrow 1$ . Physically this is because that in this case [1],  $\lim_{d_{\mathcal{U}} \rightarrow 1^+} A_{d_{\mathcal{U}}} \theta(p^2) / p^{2(2-d_{\mathcal{U}})} = 2\pi \delta(p^2)$ , the unparticle behaves as a massless particle. When  $m_p^2 \neq 0$ , the decay rate  $\Gamma(P \rightarrow \mathcal{U})$  is zero.

For  $Z \rightarrow \mathcal{U}$ , we have

$$\begin{aligned} \Gamma(Z \rightarrow \mathcal{U}^\mu, \lambda'_{\text{bO}}) &= \frac{\Lambda_{\mathcal{U}}^2}{2m_Z} \sin^2 \theta_W (\lambda'_{\text{bO}})^2 \left( \frac{m_Z^2}{\Lambda_{\mathcal{U}}^2} \right)^{d_{\mathcal{U}}} A_{d_{\mathcal{U}}}, \\ \Gamma(Z \rightarrow \mathcal{U}^\mu, \lambda'_{\text{hh}}) &= \frac{\Lambda_{\mathcal{U}}^2}{2m_Z} \frac{4\pi\alpha}{\sin^2(2\theta_W)} (\text{Im}(\lambda'_{\text{hh}}))^2 \left( \frac{v^2}{m_Z^2} \right)^2 \left( \frac{m_Z^2}{\Lambda_{\mathcal{U}}^2} \right)^{d_{\mathcal{U}}} A_{d_{\mathcal{U}}}. \end{aligned} \quad (2.6)$$



**Figure 1:** Bounds on parameter space of  $\Lambda_{\mathcal{U}}$  vs.  $\lambda'_{bO}$  allowing unparticle decay mode to saturate the difference of 3 MeV for invisible decay width of  $Z$  between SM prediction and experimental data. The lines from top to bottom are for  $d_{\mathcal{U}} = 1.3, 1.5, 2.0$ .

In general four parameters are needed to describe unparticle interactions with SM particles:  $C_{\mathcal{U}}$ ,  $\lambda$ ,  $\Lambda_{\mathcal{U}}$  and  $d_{\mathcal{U}}$  as shown in eqs. (1.1) and (1.2). If  $d_{\text{SM}} + d_{\text{BZ}} > 4$ , the parameters  $\lambda$  is less than 1. The parameter  $C_{\mathcal{U}}$  contains information about the original heavy particle mediating interaction of the SM and scale invariant sectors, and also information about the transmutation. One may normalize the parameter  $C_{\mathcal{U}}$  into the definition of  $\lambda$  for one operator, but in general will not be able to do so for more than one operator. The values for  $C_{\mathcal{U}}$  depend on the detailed dynamics. If one is only concerned with the scale where different transitions have happened, one usually sets  $C_{\mathcal{U}}$  to be one and use the two parameters  $\lambda$  and  $\Lambda_{\mathcal{U}}$  to describe the situation. In our numerical discussions, we will also follow this subscription.

Precise experimental data have been obtained on  $Z$  decay widths [29] with the invisible width to be:  $\Gamma(Z \rightarrow \text{invisible}) = 499.0 \pm 1.5 \text{ MeV}$ . This is to be compared with the width of  $501.65 \pm 0.11 \text{ MeV}$  from SM prediction for  $Z$  decay into neutrinos. New contribution to invisible  $Z$  decay is therefore constrained severely, basically needs to be within the range of experimental error bar of order one MeV. In figure 1 we show constraints on unparticle interactions allowing the unparticle contribution to invisible  $Z$  decay to saturate  $2\sigma$  error of experimental data of 3 MeV. Numerically the bound on  $Im(\lambda'_{hh})$  is about 5.5 times stronger than  $\lambda'_{bO}$  for given  $d_{\mathcal{U}}$  and  $\Lambda_{\mathcal{U}}$ . In figure 1 we only show constraint on  $\lambda'_{bO}$ . Since for a given  $d_{\mathcal{U}}$ , the scale  $\Lambda_{\mathcal{U}}$  also depends on the parameter  $\lambda'_{bO}$ , one can view the constraints on  $\lambda'_{bO}$  for a given  $\Lambda_{\mathcal{U}}$  or on the scale  $\Lambda_{\mathcal{U}}$  for a given  $\lambda'_{bO}$ . In any case, from figure 1 it is clear that the constraints are very sensitive to the dimension parameter  $d_{\mathcal{U}}$ . If  $d_{\mathcal{U}}$  is close to 1, for example with  $\lambda'_{bO} = 1$  and  $d_{\mathcal{U}}$  to be 1.3,  $\Lambda_{\mathcal{U}}$  needs to be larger than  $10^4 \text{ TeV}$ , but  $\Lambda_{\mathcal{U}}$  can be as low as one TeV for  $d_{\mathcal{U}} = 2$ . If by some means the scale  $\Lambda_{\mathcal{U}}$  is known, for example  $\Lambda_{\mathcal{U}} = 1 \text{ TeV}$ , we have the upper bounds  $\lambda'_{bO} = 0.049$  and  $0.10$  for the cases  $d_{\mathcal{U}}=1.3$  and  $1.5$ , respectively.

### 3. Constraints from invisible decay of quarkonia

For a vector quarkonium  $V$  decays into an unparticle  $\mathcal{U}$  the following operators will contribute.

$$\lambda'_{\text{QQ}}\Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}}\bar{Q}_L\gamma_{\mu}Q_L O_{\mathcal{U}}^{\mu}, \lambda'_{\text{UU}}\Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}}\bar{U}_R\gamma_{\mu}U_R O_{\mathcal{U}}^{\mu}, \lambda'_{\text{DD}}\Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}}\bar{D}_R\gamma_{\mu}D_R O_{\mathcal{U}}^{\mu}. \quad (3.1)$$

The matrix elements for vector quarkonia and unparticle transition resulting from the above interactions can be written as the following

$$M(V \rightarrow \mathcal{U}^{\mu}, \lambda') = \frac{1}{2}\lambda'\Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}}\langle 0|\bar{q}\gamma_{\mu}q|V\rangle \cdot \epsilon_{\mathcal{O}}^{\mu}, \quad (3.2)$$

where for  $q$  being an up type quark,  $\lambda'$  can be  $\lambda'_{\text{QQ}}$  and  $\lambda'_{\text{UU}}$  with  $Q_q = 2/3$ , and for  $q$  being a down type quark,  $\lambda'$  can be  $\lambda'_{\text{QQ}}$  and  $\lambda'_{\text{DD}}$  with  $Q_q = -1/3$ .

We obtain

$$\frac{Br(V \rightarrow \mathcal{U}^{\mu}, \lambda')}{Br(V \rightarrow \mu^+\mu^-)} = \frac{3A_{d_{\mathcal{U}}}\lambda'^2}{32\pi\alpha^2 Q_q^2} \left(\frac{m_V^2}{\Lambda_{\mathcal{U}}^2}\right)^{d_{\mathcal{U}}-1}. \quad (3.3)$$

The operator  $\lambda'_{\text{bO}}\Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}}B_{\mu\nu}\partial^{\mu}O^{\nu}$  also contributes to this process. We have

$$\frac{Br(V \rightarrow \mathcal{U}^{\mu}, \lambda'_{\text{bO}})}{Br(V \rightarrow \mu^+\mu^-)} = \frac{3A_{d_{\mathcal{U}}}\cos^2\theta_W|\lambda'_{\text{bO}}|^2}{2\alpha} \left(\frac{m_V^2}{\Lambda_{\mathcal{U}}^2}\right)^{d_{\mathcal{U}}-1}. \quad (3.4)$$

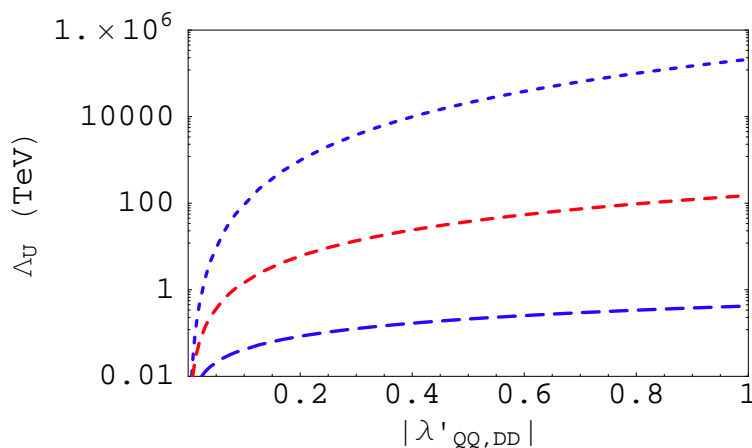
In figure 2 we show constraints on unparticle interactions using experimental data [28]:  $Br(\Upsilon \rightarrow \text{invisible}) < 2.5 \times 10^{-3}$ . For this case  $Q = D = b$  and  $Q_q = -1/3$ . In obtaining the constraints, we have neglected small contributions from  $\Upsilon \rightarrow \nu\bar{\nu}$  to  $\Upsilon$  invisible decay width. In this case, for given  $d_{\mathcal{U}}$  and  $\Lambda_{\mathcal{U}}$ , the constraints on  $\lambda'_{\text{QQ}}$  and  $\lambda'_{\text{DD}}$  are the same, while the constraint on  $\lambda'_{\text{bO}}$  is 5.6 times weaker than the bounds for  $\lambda'_{\text{QQ,DD}}$ . In figure 2, we show constraints on  $\lambda'_{\text{QQ,DD}}$  and  $\Lambda_{\mathcal{U}}$ . Again we see that the bounds are very sensitive to the dimension  $d_{\mathcal{U}}$ . If  $d_{\mathcal{U}}$  is close to 1, with  $\lambda'_{\text{QQ}} = 1$  and  $d_{\mathcal{U}} = 1.3$ ,  $\Lambda_{\mathcal{U}}$  needs to be larger than  $2 \times 10^5$  TeV, but  $\Lambda_{\mathcal{U}}$  can be as low as 400 GeV for  $d_{\mathcal{U}} = 2$ . If the scale  $\Lambda_{\mathcal{U}}$  is set to be 1 TeV, we find the upper bounds  $\lambda'_{\text{QQ,DD}} = 0.025$  and  $0.082$  for the cases  $d_{\mathcal{U}}=1.3$  and  $1.5$  respectively. Note that the constraint on  $\lambda'_{\text{bO}}$  obtained from  $\Upsilon \rightarrow \mathcal{U}$  is weaker than that obtained from  $Z \rightarrow \mathcal{U}$  by a factor of  $1.49(m_Z/m_{\Upsilon})^{d_{\mathcal{U}}-1}$ , which is in the range  $2.9 \sim 14.4$  when  $d_{\mathcal{U}}$  is in the range of  $1.3 \sim 2.0$ .

One can easily work out the case for invisible decay of  $J/\psi$  by taking  $Q = c$ ,  $U = c$  and  $Q_q = 2/3$ . BES has accumulated more than a million  $J/\psi$ , it would be interesting to see if these data when analyzed for invisible decay, a better constraint could be obtained.

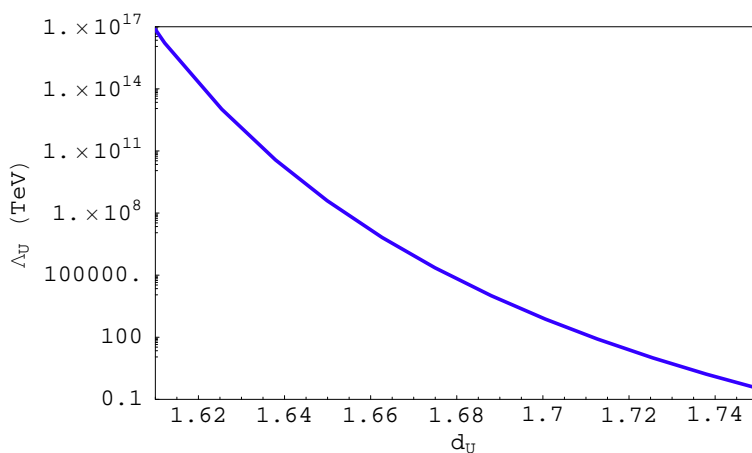
### 4. Constraints from invisible decay of neutrinos

For an active neutrino decays into an unparticle, the following operator will contribute

$$\lambda_s\Lambda_{\mathcal{U}}^{3/2-d_{\mathcal{U}}}\bar{L}_L H O_{\mathcal{U}}^s. \quad (4.1)$$



**Figure 2:** Bounds on parameter space of  $\Lambda_U$  vs.  $\lambda'_{QQ,DD}$  allowing unparticle decay mode to saturate the experimental data for invisible  $\Upsilon$  decay. The lines from top to bottom are for  $d_U = 1.3, 1.5, 2.0$ .



**Figure 3:** Allowed parameter space for  $\Lambda_U$  vs.  $d_U$ .

After the Higgs develops its vev, one obtains a transition matrix element between a neutrino and an unparticle

$$M(\nu \rightarrow \mathcal{U}^s) = \lambda_s \Lambda_U^{3/2-d_U} \bar{\nu}_L \frac{v}{\sqrt{2}} O_U^s. \quad (4.2)$$

This leads to

$$\Gamma(\nu \rightarrow \mathcal{U}^s) = \frac{1}{4} |\lambda_s|^2 \frac{v^2}{m_\nu} \left( \frac{m_\nu^2}{\Lambda_U^2} \right)^{d_U-3/2}. \quad (4.3)$$

Using constraints from solar neutrino data [33]  $\tau/m > 10^{-4} \text{s/eV}$  on neutrino lifetime and mass ratio, one can obtain information about the unparticle interactions. Since the

term generating this invisible neutrino decay is related to the Yukawa coupling, it is natural to have  $\lambda_s v/\sqrt{2}$  to be of order the neutrino mass itself (one can easily convert into different normalization). In this case one would obtain

$$m_\nu^2 \left( \frac{m_\nu^2}{\Lambda_{\mathcal{U}}^2} \right)^{d_{\mathcal{U}}-3/2} < 1.3 \times 10^{-11} \text{eV}^2. \quad (4.4)$$

Applying this formula to the solar neutrino with the constraint on its relevant mass to be larger than  $\sqrt{\Delta m_{\text{solar}}^2}$ , we obtain the bound on  $\Lambda_{\mathcal{U}}$  as a function of  $d_{\mathcal{U}}$  in figure 3 with the central value of  $\Delta m_{\text{solar}}^2 = 8.0 \times 10^{-5} \text{eV}^2$  [29]. It can be easily seen that the bound on  $\Lambda_{\mathcal{U}}$  depends on  $d_{\mathcal{U}}$  very sensitively. If one sets  $\Lambda_{\mathcal{U}}$  to be less than the Planck scale  $m_P = 1.22 \times 10^{19} \text{GeV}$ , the dimension  $d_{\mathcal{U}}$  must be bigger than 1.6.

## 5. Constraints from radiative $V \rightarrow \gamma + \text{invisible}$

Now we study the possibility of using  $V(Z) \rightarrow \gamma + \mathcal{U}$  to constrain the unparticle interactions. Using the general formula in eq. (2.3), we obtain the differential rate for  $V \rightarrow \gamma + \mathcal{U}$  for a given matrix element  $M$ ,

$$\frac{d\Gamma(P \rightarrow \gamma + \mathcal{U})}{dE_\gamma} = \frac{|\overline{M}|^2}{2m_P} A_{\text{du}}(P_{\mathcal{U}}^2)^{d_{\mathcal{U}}-2} \frac{E_\gamma}{4\pi^2}. \quad (5.1)$$

For  $Z(V) \rightarrow \mathcal{U}$  process, the unparticle  $\mathcal{U}$  must be a vector type. For  $Z(V) \rightarrow \gamma + \mathcal{U}$ , the unparticle can be a scalar or a vector. Using this process, constraints on scalar unparticle interactions can also be obtained. We find that the following operators contribute to  $V \rightarrow \gamma + \mathcal{U}$  at the tree level,

$$\begin{aligned} \text{(a)} \quad & \lambda_{\text{ww}} \Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}} W^{\mu\nu} W_{\mu\nu} O_{\mathcal{U}}, \quad \lambda_{\text{bb}} \Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}} B^{\mu\nu} B_{\mu\nu} O_{\mathcal{U}}, \quad \tilde{\lambda}_{\text{ww}} \Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}} \tilde{W}^{\mu\nu} W_{\mu\nu} O_{\mathcal{U}}, \\ & \tilde{\lambda}_{\text{bb}} \Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}} \tilde{B}^{\mu\nu} B_{\mu\nu} O_{\mathcal{U}}, \\ \text{(b)} \quad & \lambda_{\text{QQ}} \Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}} \bar{Q}_L \gamma_\mu D^\mu Q_L O_{\mathcal{U}}, \quad \lambda_{\text{UU}} \Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}} \bar{U}_R \gamma_\mu D^\mu U_R O_{\mathcal{U}}, \quad \lambda_{\text{DD}} \Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}} \bar{D}_R \gamma_\mu D^\mu D_R O_{\mathcal{U}}, \\ \text{(c)} \quad & \tilde{\lambda}_{\text{QQ}} \Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}} \bar{Q}_L \gamma_\mu Q_L \partial^\mu O_{\mathcal{U}}, \quad \tilde{\lambda}_{\text{UU}} \Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}} \bar{U}_R \gamma_\mu U_R \partial^\mu O_{\mathcal{U}}, \quad \tilde{\lambda}_{\text{DD}} \Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}} \bar{D}_R \gamma_\mu D_R \partial^\mu O_{\mathcal{U}}, \\ \text{(d)} \quad & \lambda'_{\text{QQ}} \Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}} \bar{Q}_L \gamma_\mu Q_L O_{\mathcal{U}}^\mu, \quad \lambda'_{\text{UU}} \Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}} \bar{U}_R \gamma_\mu U_R O_{\mathcal{U}}^\mu, \quad \lambda'_{\text{DD}} \Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}} \bar{D}_R \gamma_\mu D_R O_{\mathcal{U}}^\mu, \\ & \lambda'_{\text{bO}} \Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}} B_{\mu\nu} \partial^\mu O^\nu. \end{aligned} \quad (5.2)$$

For class (a) contributions, we obtain

$$\frac{Br(V \rightarrow \gamma + \mathcal{U}, \lambda_{\text{GG}})}{Br(V \rightarrow \mu^+ \mu^-)} = \int dE_\gamma \frac{A_{d_{\mathcal{U}}} E_\gamma^3 (\lambda_{\text{GG}})^2}{\pi^2 \alpha (\Lambda_{\mathcal{U}}^2)^{d_{\mathcal{U}}} (m_V^2 - 2m_V E_\gamma)^{2-d_{\mathcal{U}}}}, \quad (5.3)$$

where  $\lambda_{\text{GG}}$  takes the values  $\lambda_{\text{ww}} \sin^2 \theta_W$ ,  $\lambda_{\text{bb}} \cos^2 \theta_W$ ,  $\tilde{\lambda}_{\text{ww}} \sin^2 \theta_W$  and  $\tilde{\lambda}_{\text{bb}} \cos^2 \theta_W$  for the four operators in class (a) in order, respectively.

For classes (b) and (c) contributions, we have

$$\begin{aligned} \frac{Br(V \rightarrow \gamma + \mathcal{U}, \lambda)}{Br(V \rightarrow \mu^+ \mu^-)} &= \int dE_\gamma \frac{A_{d_{\mathcal{U}}} m_V^2 E_\gamma \lambda^2}{8\pi^2 \alpha (\Lambda_{\mathcal{U}}^2)^{d_{\mathcal{U}}} (m_V^2 - 2m_V E_\gamma)^{2-d_{\mathcal{U}}}}, \\ \frac{Br(V \rightarrow \gamma + \mathcal{U}, \tilde{\lambda})}{Br(V \rightarrow \mu^+ \mu^-)} &= \int dE_\gamma \frac{A_{d_{\mathcal{U}}} m_V^2 E_\gamma \tilde{\lambda}^2}{4\pi^2 \alpha (\Lambda_{\mathcal{U}}^2)^{d_{\mathcal{U}}} (m_V^2 - 2m_V E_\gamma)^{2-d_{\mathcal{U}}}}. \end{aligned} \quad (5.4)$$

For class (d), contributions from the first three operators are give by

$$\frac{Br(V \rightarrow \gamma + \mathcal{U}, \lambda')}{Br(V \rightarrow \mu^+ \mu^-)} = \int dE_\gamma \frac{A_{d_{\mathcal{U}}}(m_V^2 - m_V E_\gamma) E_\gamma \lambda'^2}{2\pi^2 \alpha (\Lambda_{\mathcal{U}}^2)^{d_{\mathcal{U}}-1} (m_V^2 - 2m_V E_\gamma)^{3-d_{\mathcal{U}}}}. \quad (5.5)$$

In the above  $\lambda = \lambda_{\text{QQ}}$  or  $\lambda_{\text{DD}}$  and  $\lambda = \lambda_{\text{QQ}}$  or  $\lambda_{\text{DD}}$  for quarkonia composed of down and up type of quarks, respectively. Similarly for  $\tilde{\lambda}$  and  $\lambda'$ .

The fourth operator  $\lambda'_{\text{bO}} \Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}} B_{\mu\nu} \partial^\mu O^\nu$  in class (d) also contributes to  $V \rightarrow \gamma + \mathcal{U}$  and the decay width can be obtained by replacing  $\lambda'$  with  $\lambda'_{\text{bO}}(eQ_q \cos \theta_W)$  in eq. (5.5).

It is interesting to note that for contributions from classes (a), (b) and (c),  $d_{\mathcal{U}}$  needs to be larger than 1 in order to have a finite width for  $V \rightarrow \gamma + \mathcal{U}$ , while for the contributions from class (d),  $d_{\mathcal{U}}$  needs to be larger than 2 to have a finite width. In our numerical analysis, we will let  $d_{\mathcal{U}}$  to be larger than 2 for this case. Also note that the distributions of  $E_\gamma$  for class (a), classes (b) and (c), and class (d) are different. This can be used to distinguish different contributions if enough data are accumulated.

There are experimental constrains on  $\Upsilon$  and  $J/\psi$  with  $Br(J/\psi \rightarrow \gamma + \text{invisible}) < 1.4 \times 10^{-5}$  [30], and  $Br(\Upsilon(1S) \rightarrow \gamma + \text{invisible}) < 1.5 \times 10^{-5}$  [31, 32]. Combining the formula obtained above for unparticle contributions, one can set constraints on unparticle interactions. The results are shown in figure 4. We find that the present upper bounds on  $\Upsilon(J/\psi) \rightarrow \gamma + \text{invisible}$  decay widths do not give strong constraints on the unparticle interactions. Improved bounds can provide more information.

We comment that there are also contributions to  $Z \rightarrow \gamma + \mathcal{U}$  from class (a) operators. The decay width is given by

$$\frac{d\Gamma}{dE_\gamma} = \frac{A_{d_{\mathcal{U}}} \sin^2(2\theta_w)}{3\pi^2} \frac{E_\gamma^3 m_Z}{(m_Z^2 - 2m_Z E_\gamma)^2} \left( \frac{m_Z^2 - 2m_Z E_\gamma}{\Lambda_{\mathcal{U}}^2} \right)^{d_{\mathcal{U}}} \lambda^2, \quad (5.6)$$

where  $\lambda$  can be any of  $\lambda_{ww,bb}, \tilde{\lambda}_{ww,bb}$ . It would be interesting to see if strong constraints can be obtained for unparticle interactions when LEP data are analyzed for  $Z \rightarrow \gamma + \text{invisible}$ .

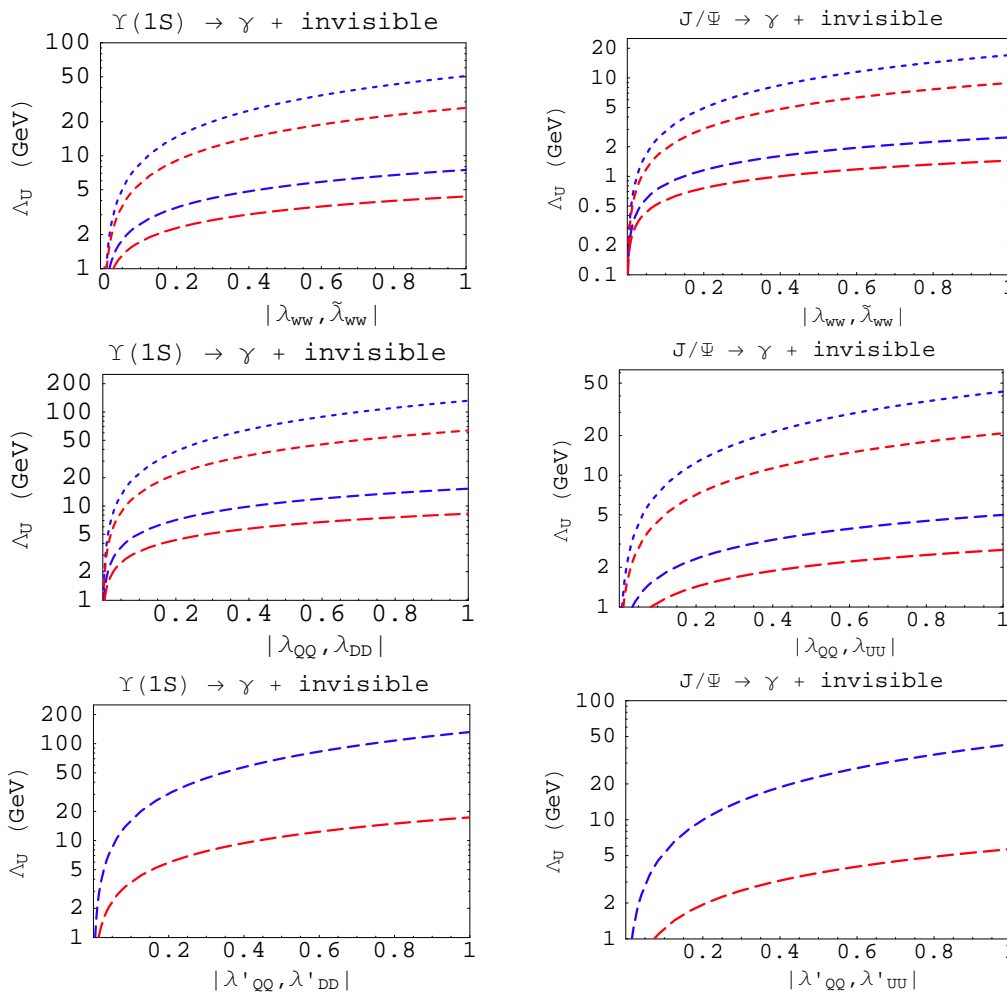
## 6. Summary

If unparticles exist they must interact weakly with particles. The direct signature of unparticles will be in the form of missing energy in decays and collisions of particles. In this paper we have studied constraints on unparticle interactions using totally invisible decay modes of  $Z$ , vector quarkonia  $V$  and neutrinos. There are several operators which can contribute to these decays.

Two operators with couplings  $\lambda'_{\text{hh}}$  and  $\lambda'_{\text{bO}}$  contribute to  $Z \rightarrow \mathcal{U}$ . Numerically the bound on  $Im(\lambda'_{\text{hh}})$  is about 5.5 times stronger than  $\lambda'_{\text{bO}}$  for given  $d_{\mathcal{U}}$  and  $\Lambda_{\mathcal{U}}$ . The constraints are very sensitive to the dimension parameter  $d_{\mathcal{U}}$ . If  $d_{\mathcal{U}}$  is close to 1, with  $\lambda'_{\text{bO}} = 1$  and  $d_{\mathcal{U}} = 1.3$ ,  $\Lambda_{\mathcal{U}}$  needs to be larger than  $10^4$  TeV, but  $\Lambda_{\mathcal{U}}$  can be as low as one TeV for  $d_{\mathcal{U}} = 2$ . If by some means that the scale  $\Lambda_{\mathcal{U}}$  is known, for example  $\Lambda_{\mathcal{U}} = 1$  TeV, we have the upper bounds  $\lambda'_{\text{bO}} = 0.049$  and  $0.10$  for the cases  $d_{\mathcal{U}}=1.3$  and  $1.5$ , respectively.

Several operators contribute to  $V \rightarrow \mathcal{U}$ , including the operator with coupling  $\lambda'_{\text{bO}}$  and additional ones  $\lambda'_{\text{QQ, UU, DD}}$ . There is experimental upper bound from  $\Upsilon \rightarrow \text{invisible}$ . We





**Figure 4:** Bounds on parameter space of  $\lambda$  vs.  $\Lambda_{\mathcal{U}}$  allowing unparticle decay mode to saturate the experimental data for  $\gamma + \text{invisible}$  of  $\Upsilon$  and  $J/\psi$  decays. From top to bottom, the lines represent the cases for  $d_{\mathcal{U}} = 1.3, 1.5, 2.1, 2.5$  and for the bottom panel,  $d_{\mathcal{U}}$  only takes 2.1 and 2.5.

find that the constraints on  $\lambda'_{\text{bO}}$  are weaker than the constraint on  $\lambda'_{\text{QQ,DD}}$  by a factor of 5.6. The constraints are again sensitive to  $d_{\mathcal{U}}$ . If  $d_{\mathcal{U}}$  is close to 1, with  $\lambda'_{\text{QQ}} = 1$  and  $d_{\mathcal{U}} = 1.3$ ,  $\Lambda_{\mathcal{U}}$  needs to be larger than  $2 \times 10^5$  TeV, but  $\Lambda_{\mathcal{U}}$  can be as low as 400 GeV for  $d_{\mathcal{U}} = 2$ . If the scale  $\Lambda_{\mathcal{U}}$  is set to be 1 TeV, we find the upper bounds  $\lambda'_{\text{QQ,DD}} = 0.025$  and 0.082 for the cases  $d_{\mathcal{U}}=1.3$  and 1.5 respectively. The constraint on  $\lambda'_{\text{bO}}$  obtained from  $\Upsilon \rightarrow \mathcal{U}$  is weaker than that obtained from  $Z \rightarrow \mathcal{U}$  by a factor of  $1.49(m_Z/m_{\Upsilon})^{d_{\mathcal{U}}-1}$ , which is in the range  $2.9 \sim 14.4$  when  $d_{\mathcal{U}}$  takes value in the range of  $1.3 \sim 2.0$ .

There is one operator which can induce neutrino to  $\mathcal{U}$  decay. Strong constraint could be obtained using constraint on  $\tau/m$  obtained from solar neutrino data. If one sets  $\Lambda_{\mathcal{U}}$  to be less than the Planck scale  $m_P = 1.22 \times 10^{19}$  GeV, the dimension  $d_{\mathcal{U}}$  must be bigger than 1.6.

We also studied the possibility of using  $V(Z) \rightarrow \gamma + \mathcal{U}$  to constrain unparticle inter-

actions. We find that present experimental upper bounds for  $\Upsilon(J/\psi) \rightarrow \gamma + \text{invisible}$  does not give strong bounds on unparticle interactions.

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